The Q3D-working-range reduced QP

The reduced quadratic program that belongs to the Q3D working range system has the form

$$\min_{\Delta \boldsymbol{p}} \quad \Delta \boldsymbol{p}^\top \boldsymbol{B} \Delta \boldsymbol{p} + \boldsymbol{\gamma}^\top \Delta \boldsymbol{p} \tag{1}$$

$$\text{s. t.} \quad \begin{pmatrix} \mathbf{g} \\ \mathbf{h} \end{pmatrix} + \boldsymbol{G} \Delta \boldsymbol{p} \leq \mathbf{0} \tag{2}$$

with the linearized constraint matrix

$$\boldsymbol{G} \;=\; \begin{pmatrix} \nabla_{p_1}^\top \mathbf{g}_1 & & \\ & \ddots & \\ & & \nabla_{p_M}^\top \mathbf{g}_M \\ \nabla_{p_1}^\top \mathbf{h} & \dots & \nabla_{p_M}^\top \mathbf{h} \end{pmatrix} \tag{3}$$

and a rank-two update matrix $\boldsymbol{B}$ approximating $\boldsymbol{H}_{pp}$:

$$\boldsymbol{B} \;:=\; \boldsymbol{B}^{+}(\boldsymbol{B}, \Delta \boldsymbol{p}^{(\alpha)}, \Delta \boldsymbol{\gamma}^{+}, \rho) \; . \tag{4}$$

The linearized system of flow equations has the block diagonal structure

$$\nabla_x \mathbf{c} \;=\; \begin{pmatrix} \nabla_{x_{1,1}} \mathbf{c}_{1,1} & & & & \\ & \ddots & & & \\ & & \nabla_{x_{N_1,1}} \mathbf{c}_{x_{N_1,1}} & & \\ & & & \ddots & \\ & & & & \nabla_{x_{N_M,M}} \mathbf{c}_{x_{N_M,M}} \end{pmatrix} \tag{5}$$

so that the reduced gradient has the form

$$\boldsymbol{\gamma} \;=\; \begin{pmatrix} w_1 \, \boldsymbol{\gamma}_1 \\ \vdots \\ w_M \, \boldsymbol{\gamma}_M \end{pmatrix} \tag{6}$$

with components

$$\boldsymbol{\gamma}_j \;=\; \nabla_{p_j} f_j \;-\; \nabla_{p_j} \mathbf{c}_j^\top (\nabla_x \mathbf{c}_j^\top)^{-1} \nabla_x f_j \tag{7}$$

$$\;=\; \sum_{i=1}^{N_j} \omega_{ij} \, \boldsymbol{\gamma}_{ij} \; , \tag{8}$$

where

$$\boldsymbol{\gamma}_{ij} \;=\; \nabla_{p_j} f_{ij} \;-\; \nabla_{p_j} \mathbf{c}_{ij}^\top (\nabla_{x_{ij}} \mathbf{c}_{ij}^\top)^{-1} \nabla_{x_{ij}} f_{ij} \; , \tag{9}$$

which is just the reduced gradient of one single streamline operating point.

As the reduced gradient and the cost functional (which is just the weighted sum of all single streamline operating point costs) and the update of the flow variables (where we have separate variables for each streamline and operating point), this means that we can do the flow computations at every streamline and operating point independently from each other.

This is *very* important as it allows us to keep the simple interface between optimization and flow computation as we have it at one single streamsurface operating point computation where the we use one single `ISES` iteration step to compute our projection step and the reduced gradient.

Please note that this algorithmic concept does not allow to include state constraints into the optimization system. It is, however, still possible to include constraints of that type into the objective function via a penalty or barrier approach.