Journal of Computational and Applied Mathematics 120 (2000) 59–65

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.nl/locate/cam

Working range optimization for turbine and compressor blading

Wolfgang Egartner

Interdisciplinary Center for Scientific Computing, University of Heidelberg, Im Neuenheimer Feld 368, D-69120 Heidelberg, Germany

Received 22 January 1998; received in revised form 20 December 1999

Abstract

A partially reduced SQP algorithm for a multiple setpoint industrial design optimization problem is presented. It is highlighted that the modularity of this concept has made it possible to apply it to problems as complex as working range optimization in high-temperature gas turbine and compressor blade design. Further emphasis is drawn on parallelization aspects and practical experiences in engineering practice. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Multiple setpoint optimization; Partially reduced SQP methods; Shape optimization; Transonic cascade flow; Turbine and compressor S1-streamsurface blade design

1. Introduction

In turbine and compressor design, highly expensive working power is invested in order to find stable and efficient configurations for machine working ranges. One of the cornerstones of this job is the so-called S1-streamsurface blading where blades are built up by families of two-dimensional blade profiles. These profiles have then to be optimized in their working ranges taking into account constraints arising from aerodynamic, heat transfer, aeromechanical, mechanical and manufacturing considerations.

It comes in natural to formulate this as a constrained optimization problem. A more difficult task, however, is to find a practical solution approach. Due to the huge computational effort needed for flow simulation, "black box" algorithms are prohibitively expensive. On the other hand, simultaneous optimization, i.e., a direct attack on the optimality conditions, means that the optimization algorithm

E-mail address: wolfgang.egartner@iwr.uni-heidelberg.de (W. Egartner).

PII: S0377-0427(00)00303-4

has to be merged into the blade design system leading to very high costs for software implementation and support.

At this point, the partially reduced SQP (PRSQP) method comes into play. While being a simultaneous optimization approach, it supports modular implementation in that its optimization step is split into projections towards solution and optimum.

As to working range solution, there is another advantage: the projection steps towards the working range flow solution are just the sum of the projections towards the operating point solutions. That means the computationally expensive steps of the diverse operating points can be computed in parallel.

Finally, a major problem of optimizing complex simulation and design systems is that the value of the objective function usually is not feasible (or even computable) everywhere due to model restrictions. This can lead to problems for simultaneous optimization algorithms as there is no direct information about feasibility during iteration. The PRSQP setup, however, gives some control on that subject in that additional projections towards the flow solution can be inserted any time and immediate measures like step relaxation can be taken.

The software discussed in this paper applies a combined strategy to cope with the feasibility problem:

- All available a priori knowlegde about the territory of feasibility is mathematically formulated in terms of geometry constraints.
- Additional projections towards the flow solution are used to decide whether the most recent optimization step is to be relaxed due to feasibility problems.

This paper starts out with a short overview about the forward system, practical design requirements and degrees of freedom. In Section 3, the mathematical working range problem is formulated. The general algorithmic concept is shown in Section 4. Section 5 highlights parallelization aspects and the concluding section reports about experiences in practical industrial design with a short outlook on future extensions regarding quasi-3D optimization.

2. Design system and geometry constraints

The forward solution is performed by the quasi-3D cascade design system MISES (multiple blade interacting streamtube Euler solver). It is the method of choice as it is rather accurate with respect to loading and loss (rotation and streamtube effects are incorporated, as well) while being substantially faster than comparable Navier–Stokes solvers. Consequently, it is widely used in engineering design practice where it has proven to be a valuable and flexible tool for subsonic and transonic turbine and compressor blading. A comprehensive overview about the underlying models is given in [12].

Numerically, it is a fully coupled, conservative, streamline-based inviscid quasi-3D Euler system coupled with integral boundary layer equations characterizing the viscous flowfield [3,4]. The resulting nonlinear equations are discretized on streamline-aligned structured grids and attacked by Newton iteration. The linear systems of each Newton step are solved directly [2,6].

In professional design systems, blade profile coordinates typically consist of several quintic polynomial segments, where curve tangent slopes as well as curvature are supposed to be continuous at the segment joints. For convenience, we chose a subspace spanned by B-spline parameters of order 6 with triple knots at the segment joints.

As already mentioned in the introduction, geometry conditions include design and model feasibility constraints. The latter appear rather simple, but a lot of numerical experiments with a variety of configurations are needed in order to arrive at a reasonable setup. Design constraints are even more tricky to formulate. There are still discussions with engineers about how the needs arising from aerodynamic, solidity, manufacturing and heat engineering considerations should be formulated by means of area, thickness, curvature and quasi-outlet-flow-angle constraints.

As to problem size, the grid refinement one needs for blade design is case dependent and ranges from about 150×20 to 300×40. So there are about 10 000 to 40 000 variables per operating point. On the other hand, we only have 24 B-spline parameters representing the profile. So there is a comparatively small number of degrees of freedom, which is a situation where reduced SQP methods are known to be especially efficient.

3. Optimization problem formulation

For the mathematical formulation of the working range optimization problem, all variables of the forward problem, most of which are state variables and grid node positions (the node positions are variables as MISES is based on streamline curvature discretization), are summarized as

$$x = x_1, \dots, x_N, \quad x_i \in \mathbb{R}^{m_c}, \quad N \leftarrow \text{number of operating points},$$

the blade profile B-spline parameter vector is denoted $p = (p_1, \dots, p_{m_p})$ and the operating points (like inlet Mach numbers or flow angles), are represented by

$$q = q_1, \dots, q_N, \quad N \leftarrow \text{number of operating points}.$$

The objective function at one operating point is denoted $f(x_i, p)$, the discretized flow equations $\boldsymbol{c}(x_i, p, q_i)$ and the geometry constraints $\boldsymbol{g}(p)$.

We arrive at the following setup:

$$\begin{aligned}
\min_{x_{1\dots N}, p} \quad & \sum_{i=1}^{N} \omega_i f(x_i, p), \quad f : \mathbb{R}^{m_p+m_c} \to \mathbb{R}, \\
\text{s.t.} \quad & \boldsymbol{c}_1 = \boldsymbol{c}(x_1, p, q_1) = 0, \qquad \boldsymbol{c}_1 : \mathbb{R}^{m_p+m_c} \to \mathbb{R}^{m_c}, \\
& \qquad \vdots \\
& \boldsymbol{c}_N = \boldsymbol{c}(x_N, p, q_N) = 0, \qquad \boldsymbol{c}_N : \mathbb{R}^{m_p+m_c} \to \mathbb{R}^{m_c}, \\
& \boldsymbol{g}(p) \leqslant 0, \qquad \boldsymbol{g} : \mathbb{R}^{m_p} \to \mathbb{R}^{m_g},
\end{aligned} \tag{1}$$

where the MISES Newton matrices $\nabla_{x_i} \boldsymbol{c}_i$ are supposed to have full rank for all operating points.

The following notations will be used below:

$$\mathscr{F} := \sum_{i=1}^{N} \omega_i f(x_i, p), \tag{2}$$

$$\boldsymbol{C} := (\boldsymbol{c}_1, \dots, \boldsymbol{c}_N)^\top. \tag{3}$$

4. Solution by partially reduced SQP

The optimization problem (1) is attacked by means of a slightly modified quasi-Newton algorithm for the solution of the nonlinear Karush–Kuhn–Tucker equations (first-order necessary conditions for constrained nonlinear programming) of the discretized problem:

$$\nabla_{(x,p)}\mathscr{F}^\top = \nabla_{(x,p)}\boldsymbol{C}^\top\lambda + \nabla_p\boldsymbol{g}^\top\mu, \tag{4}$$

$$\operatorname{diag}(\mu)\boldsymbol{g} = 0, \quad \mu \leqslant 0, \tag{5}$$

$$\boldsymbol{C} = 0. \tag{6}$$

In the (partially) reduced setup providing the desired modularity and parallelization features, the quasi-Newton iteration step $\Delta(x, p)$ is split so that

$$\Delta x = -\nabla_x\boldsymbol{C}^{-1}\nabla_p\boldsymbol{C}\,\Delta p - \nabla_x\boldsymbol{C}^{-1}\boldsymbol{C}, \tag{7}$$

where the geometry parameter update Δp is the solution of the quadratic program

$$\begin{aligned} \min_{\Delta p} \quad & \tfrac{1}{2}\Delta p^\top B\Delta p + \gamma^\top\Delta p, \\ \text{s.t.} \quad & \nabla\boldsymbol{g}\cdot\Delta p + \boldsymbol{g} \leqslant 0 \end{aligned} \tag{8}$$

with the so-called *reduced gradient*

$$\gamma = \nabla_p\mathscr{F} - \nabla_p\boldsymbol{C}^\top(\nabla_x\boldsymbol{C}^\top)^{-1}\nabla_x f. \tag{9}$$

In this setup, the quasi-Newton update matrices B directly approximate the Hessian's projection onto the nullspace of the linearized constraints, namely

$$B \sim \nabla_{pp}\mathscr{L} - \nabla_p\boldsymbol{C}^\top\nabla_x\boldsymbol{C}^{-\top}\nabla_{xx}\mathscr{L}\nabla_x\boldsymbol{C}^{-1}\nabla_p\boldsymbol{C}, \tag{10}$$

with $\mathscr{L}$ denoting the Lagrangian of the optimization problem.

A selection of recent works about the (partially) reduced SQP approach is [1,7,10,11] while early discussions can be found in [5,8,9].

We now have an iteration

$$x_{k+1} = x_k + \alpha_k\Delta x, \tag{11}$$

$$p_{k+1} = p_k + \alpha_k\Delta p, \tag{12}$$

where the globalization parameter α_k is to be chosen in a way that a user-specified merit function is reduced. This is another, very tricky point in the practical implementation due to feasibility problems of the forward problem, especially at the nasty off-design operating points.

5. Multiple setpoint parallelization

As the working range system has the block-diagonal structure

$$\nabla_x\boldsymbol{C} = \begin{pmatrix} \nabla_{x_1}\boldsymbol{c}_1 & & & \\ & \nabla_{x_2}\boldsymbol{c}_2 & & \\ & & \ddots & \\ & & & \nabla_{x_N}\boldsymbol{c}_N \end{pmatrix} \tag{13}$$

the reduced gradient γ for the working range problem is just the sum of the reduced gradients

$$\gamma_i = \nabla_p f(x_i, p) - \nabla_p \boldsymbol{c}_i^\top (\nabla_{x_i} \boldsymbol{c}_i^\top)^{-1} \nabla_{x_i} f(x_i, p) \tag{14}$$

of the diverse operating points to

$$\gamma = \sum_{i=1}^{N} \omega_i \gamma_i. \tag{15}$$

The state variable components also can be updated separately:

$$\Delta x_i = -\nabla_x \boldsymbol{c}_i^{-1} \nabla_p \boldsymbol{c}_i \Delta p - \nabla_x \boldsymbol{c}_i^{-1} \boldsymbol{c}_i. \tag{16}$$

This means that both the expensive state variable updates (forward system solution) and sensitivity computations (adjoint forward system solution) for the diverse operating points can be computed in parallel while the blocking part of the algorithm reduces to the solution of the rather small optimization system (8).

6. Practical experiences and outlook

The optimization software based on the described concepts is currently evaluated by blade design practitioners in order to find out what kind of additional features will be needed for practical work. The goal is to use it as a tool that automatically gives solutions with more ample stable machine working ranges.

The implemented cost functionals are proven performance indicators like weighted sums over profile pressure loss coefficients at selected operating points. Solidness and heat engineering (e.g., internal cooling) requirements usually involve lower bounds on profile area, leading edge (LE) and trailing edge (TE) thicknesses. Construction constraints mainly require curvature constraints. Furthermore, a specified outlet flow angle has to be retained, which can be achieved indirectly by a special mix of geometry constraints.

Model feasibility constraints arise, for instance, from the fact that MISES profiles are open at the trailing edge (with a Kutta outflow condition applied to the endpoints), which makes way for many undesirable effects. Also, kinks at the segment joints of the leading edge segments have to be avoided a priori by means of curvature constraints.

With the constraints being carefully formulated, significant performance gains are achieved including off-design inlet flow angles and Mach numbers.

Of course, the optimization software cannot give a fully automized tool as long as there remain practical demands with no adequate mathematical a priori formulation. In this case, a lot of manual work will remain for the engineer to arrive at a useful working range solution. This is exactly the point why it is so important that the optimization is fast: it has to be used as an interactive tool.

As a comprehensive description of this complex procedure would be beyond the scope of this article, only a little snapshot is given in Fig. 1. It shows a single compressor blade profile and the corresponding total pressure loss coefficients taken at five operating points (inlet flow angles). Apart from the significant reduction in pressure loss gained by a single optimization run, one can easily see the importance of optimizing the full working range of the machine at the same time.

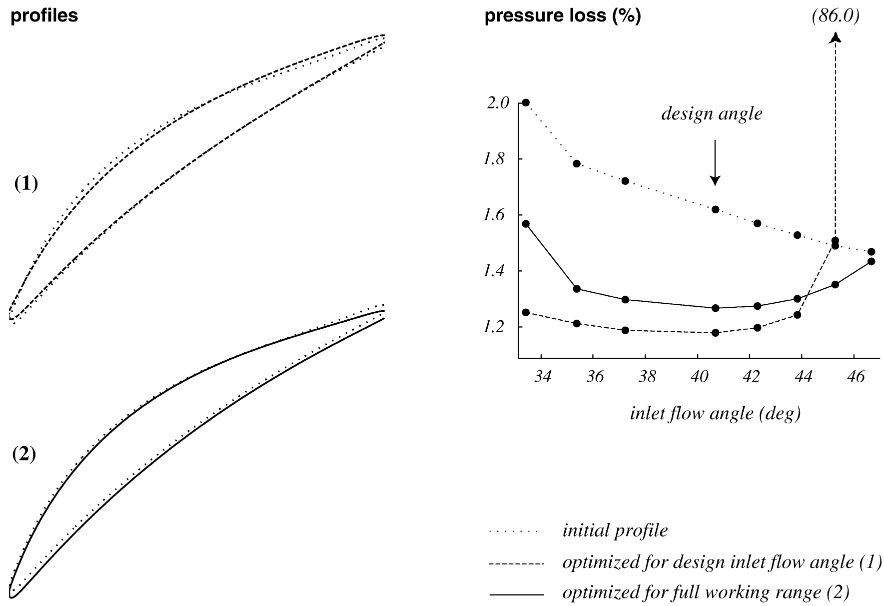

Fig. 1. Initial (dotted lines) and optimized compressor blade profiles. The profile optimized for the design angle only (dashed line) turns out to be unstable over the machine working range.

Grid resolutions in this example are slightly different for the various operating points, about 200×20 each. This run took about 15 min on a PentiumII/400-based Linux-system, which is just about 4 times the forward working range design solution.

Apart from software quality enhancement, future efforts will be focused on algorithms finding profile families allowing the interpolation of optimal 3D blades.

The algorithmic concept of this task is already worked out. While the problem will grow in size as we have ℓ blades parameterized by p_k, $m_c \times \ell$ forward systems $\boldsymbol{c}_{ik}$, $m_g \times \ell$ geometry constraint systems $\boldsymbol{g}_{jk}$ and quasi-3D geometry conditions

$$G(p_1, \ldots, p_\ell) = 0,$$

there is no need for a substantial change in the algorithms. The relatively small optimization QP will increase while the large forward and adjoint forward systems remain as they are. Their number increases, but they remain independent and can still be computed in parallel.

A more demanding and yet unresolved issue, however, is the mathematical formulation of quasi-3D conditions leading to practically useful results. Hopefully, the corresponding efforts will help to take another step towards more efficient turbomachinery design.

Acknowledgements

This project has been supported by ABB Power Generation Ltd, MTU Motoren und Turbinen-Union München GmbH and the Federal Ministry of Education, Science, Research and Technology (BMBF) under Grant Number 032741A within the German research effort AG Turbo. The project is supervised by Prof. Dr. H. G. Bock at the IWR, University of Heidelberg. Furthermore, I want to thank Volker Schulz (IWR, Prof. Wittum's group) for his help regarding PRSQP algorithms and Wolfgang Kappis (ABB Power Generation, Baden) for the generation of test cases and for many valuable hints regarding practical S1-streamsurface blading.

References

[1] L.T. Biegler, J. Nocedal, C. Schmid, A reduced Hessian method for large-scale constrained optimization, SIAM J. Optim. 5 (2) (1995) 314–347.
[2] M. Drela, M. Giles, W.T. Thompkins Jr., Newton solution of coupled Euler and boundary-layer equations. in: Numerical and Physical Aspects of Aerodynamic Flows III, Proceedings of the Third Symposium, Long Beach/California, 1986, pp. 143–154.
[3] M. Drela, Two-dimensional transonic aerodynamic design and analysis using the Euler equations, Ph.D Thesis, MIT, December 1985.
[4] M. Drela, M.B. Giles, ISES: a two-dimensional viscous aerodynamic design and analysis code, in: AIAA Conference, No. 0424, January 1987.
[5] D. Gabay, Reduced quasi-Newton methods with feasibility improvement for nonlinear constrained optimization, Math. Programming Study 16 (1982) 18–44.
[6] M.B. Giles, Newton solution of steady two-dimensional transonic flow, Ph.D Thesis, MIT, October 1985.
[7] F.-S. Kupfer, An infinite-dimensional convergence theory for reduced SQP methods in Hilbert space, SIAM J. Optim. 6 (1) (1996) 126–163.
[8] J. Nocedal, M.L. Overton, Projected Hessian updating algorithms for nonlinearly constrained optimization, SIAM J. Numer. Anal. 22 (5) (1985) 821–850.
[9] M.J.D. Powell, The convergence of variable metric methods for nonlinearly constrained optimization calculations, in: O.L. Mangasarian, R.R. Meyer, S.M. Robinson (Eds.), Nonlinear Programming, Vol. 3, Academic Press, New York, 1978, pp. 27–63.
[10] V.H. Schulz, Reduced SQP methods for large-scale optimal control problems in DAE with applications to path planning problems for satellite mounted robots, Ph.D Thesis, IWR, University of Heidelberg, Germany, 1996.
[11] V.H. Schulz, Solving discretized optimization problems by partially reduced SQP methods, Computing Visualization Sci. 1 (2) (1997).
[12] H. Youngren, Viscous design and analysis methods for transonic compressor blading, Ph.D Thesis, Gas Turbine Laboratory, Department for Aeronautics and Astronautics, MIT, February 1991.