Shape optimization of turbine and compressor blades

What is a turbomachine?

What is S1-streamsurface turbine and compressor design?

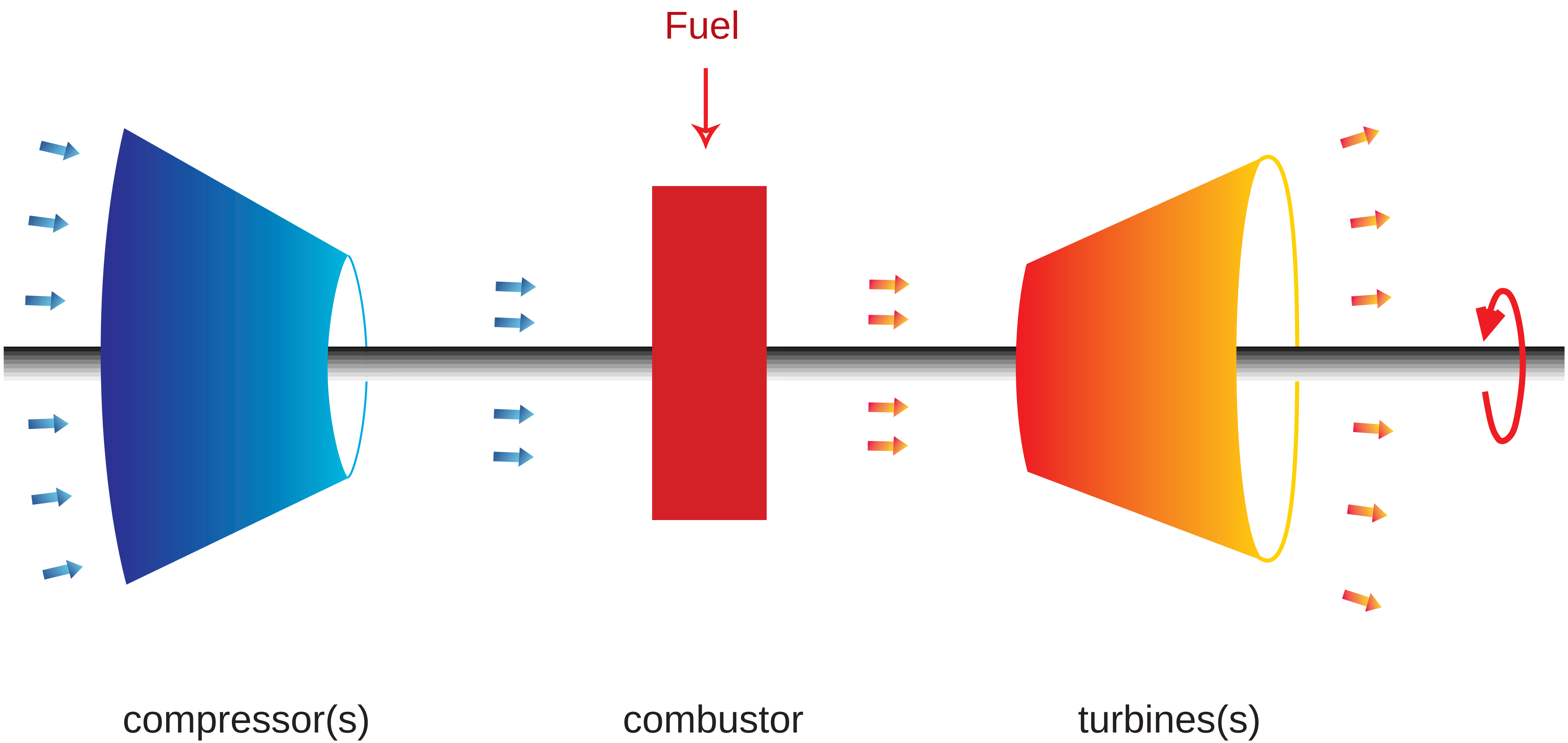
Flow separation, outlet flow angle and other nasty stuff

Implementation demands

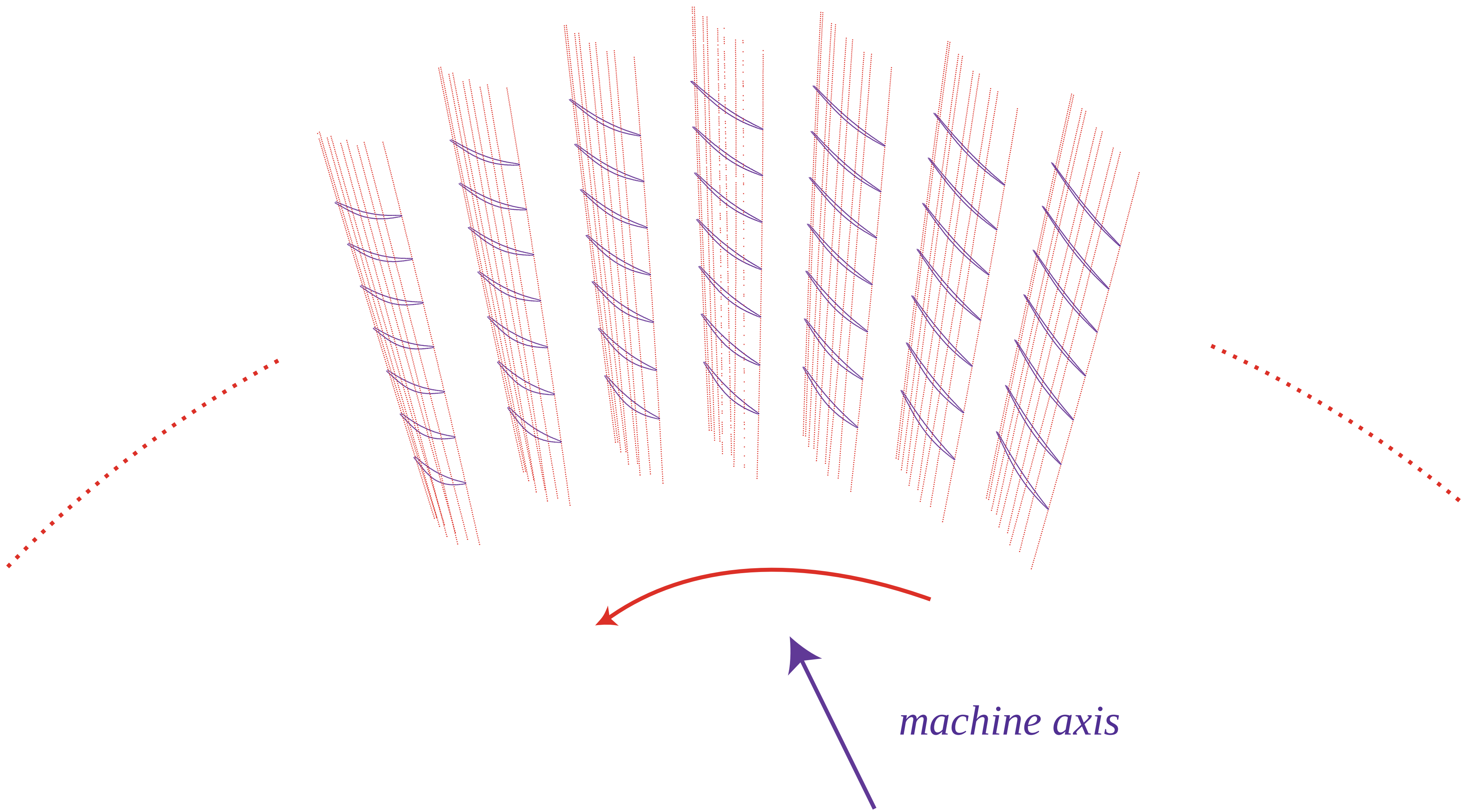
Multiple setpoint problem: working range and beautiful blades

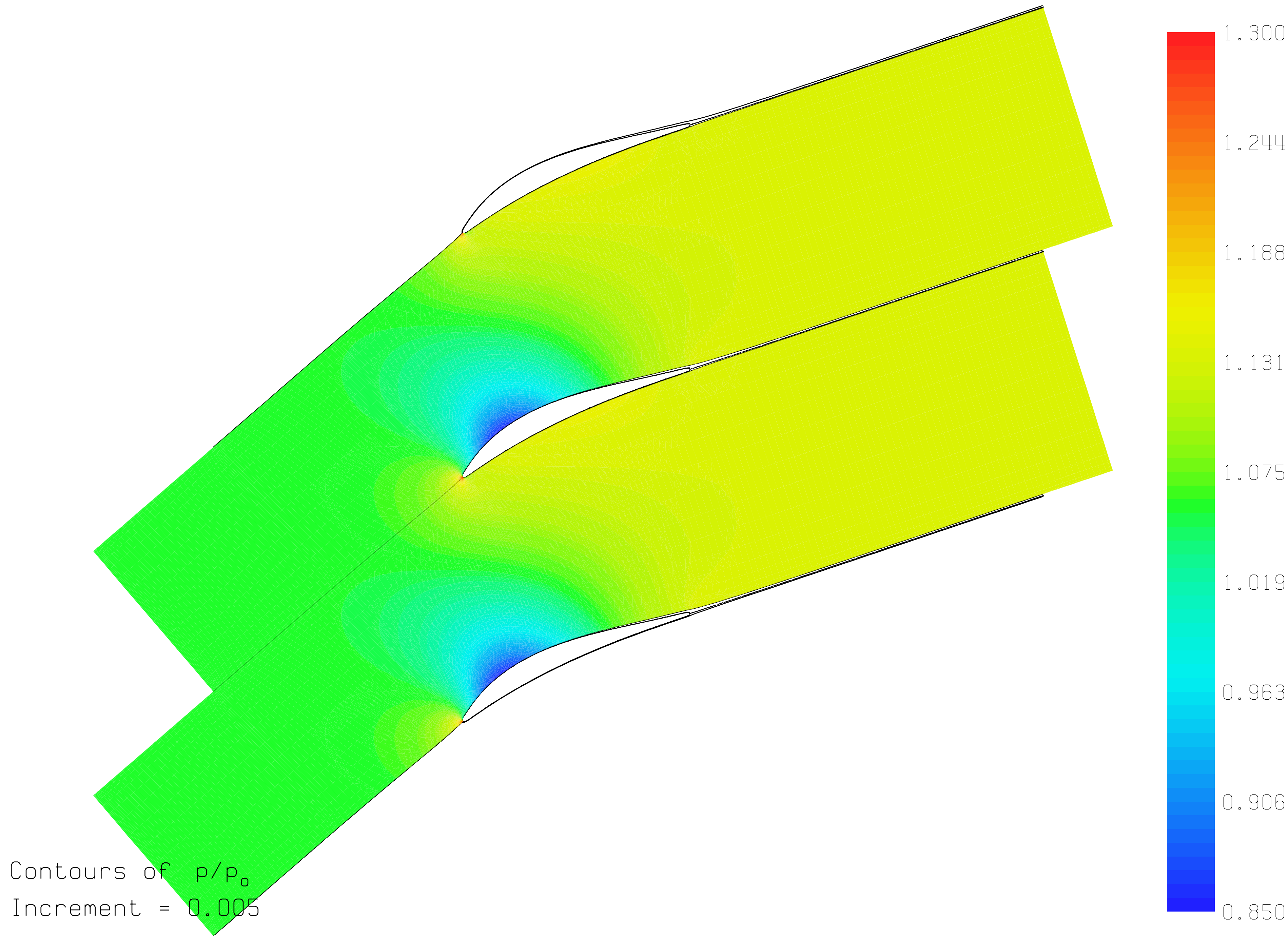
Results

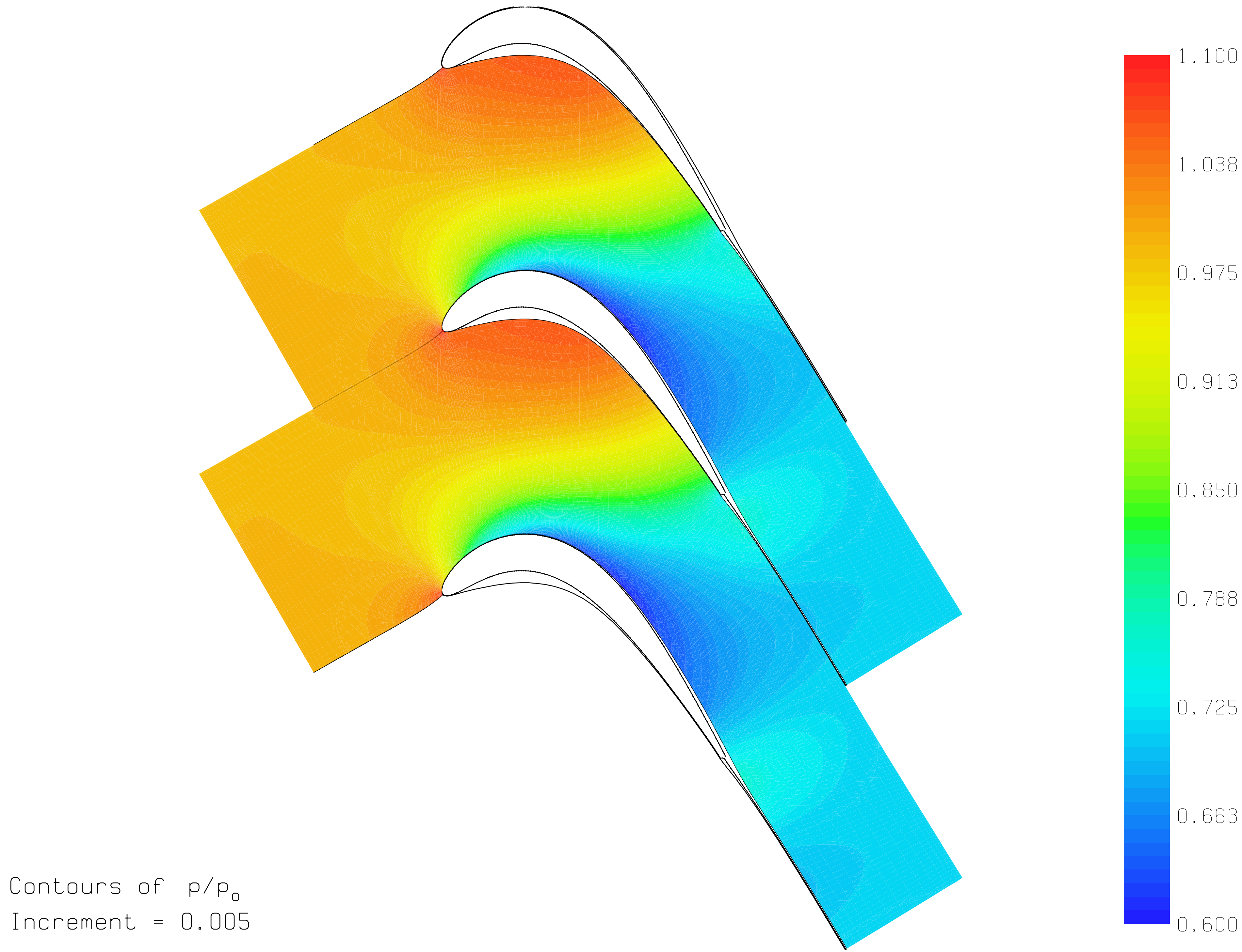
Simple turbomachine



Q3D blading




1.300
1.244
1.188
1.131
1.075
1.019
0.963
0.906
0.850
Contours of p/p_o
Increment = 0.005


1.100
1.038
0.975
0.913
0.850
0.788
0.725
0.663
0.600
Contours of p/p_o
Increment = 0.005

Engineering demands

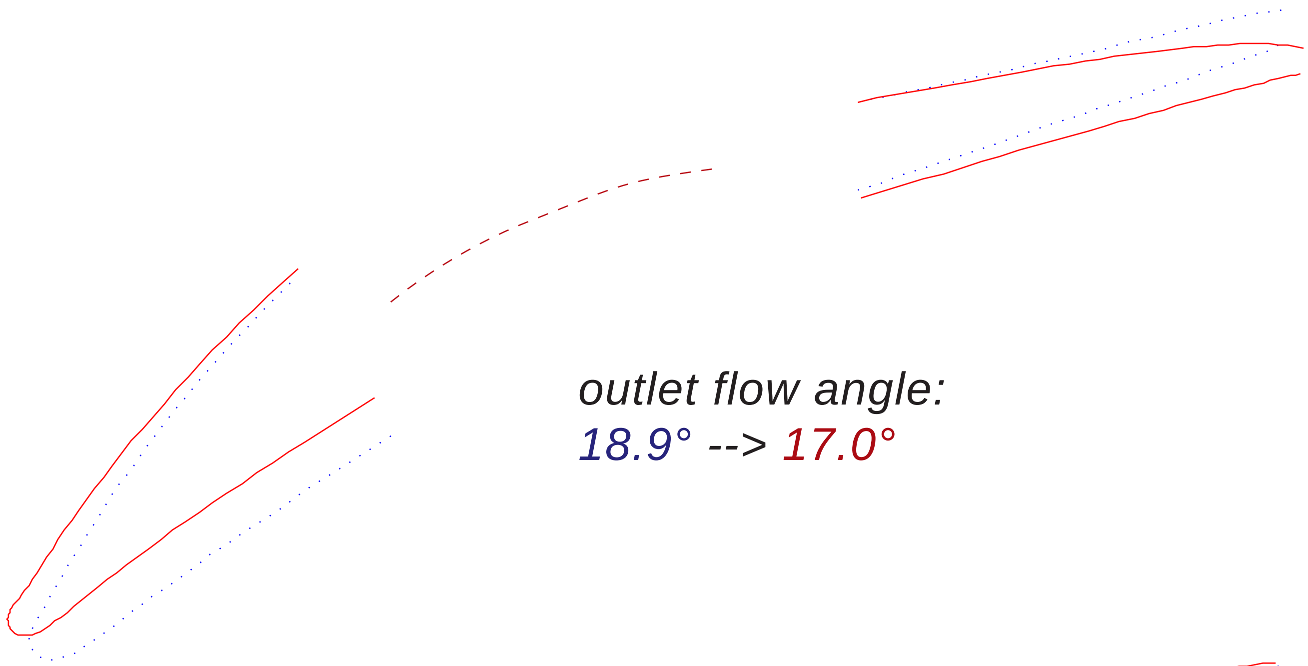
optimize performance (minimize total pressure loss, maximize efficiency)
avoid flow separation
preserve desired outflow angle (given by S2-design)
upper bound on width (construction)
lower bound on area (solidness)
upper bound on curvature (construction)
lower bound on leading edge thickness (solidness, heat engineering)
upper bound on trailing edge thickness (solidness)

Odds and ends

preserve a "beautiful" interpolated 3D-blade (avoid unwanted 3D-effects?)
... and don't let MISES down (there is no exception handling!!)

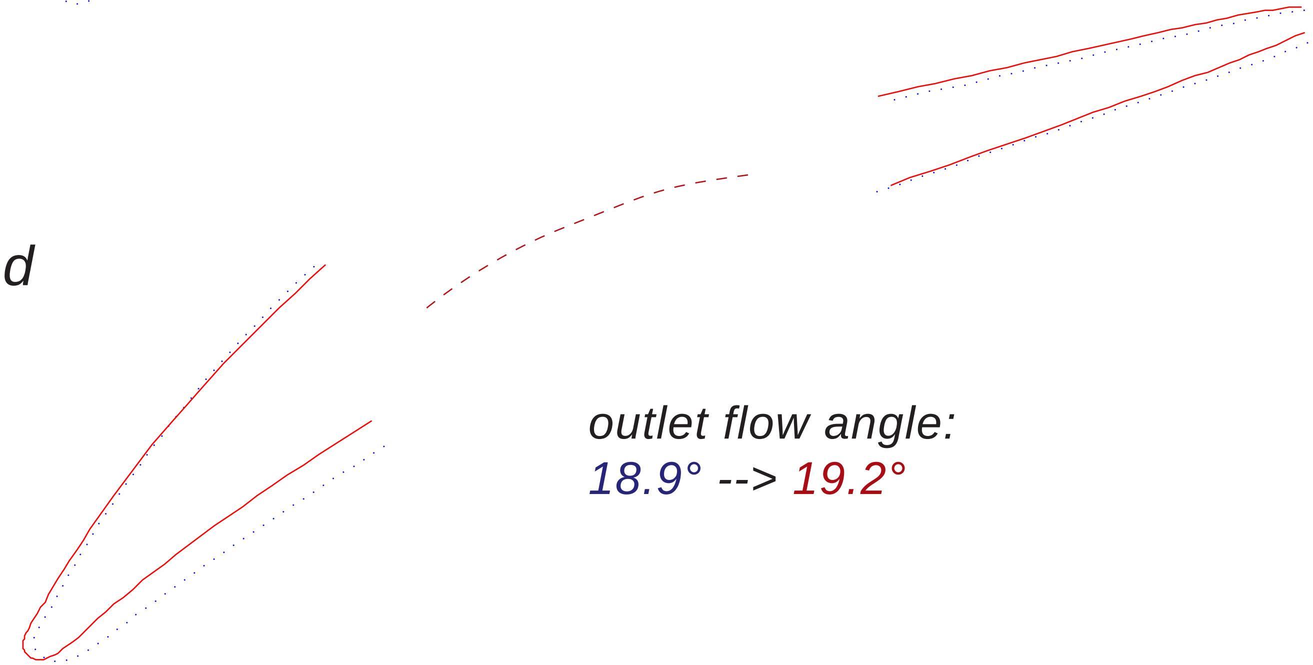
outlet flow angle

unconstrained



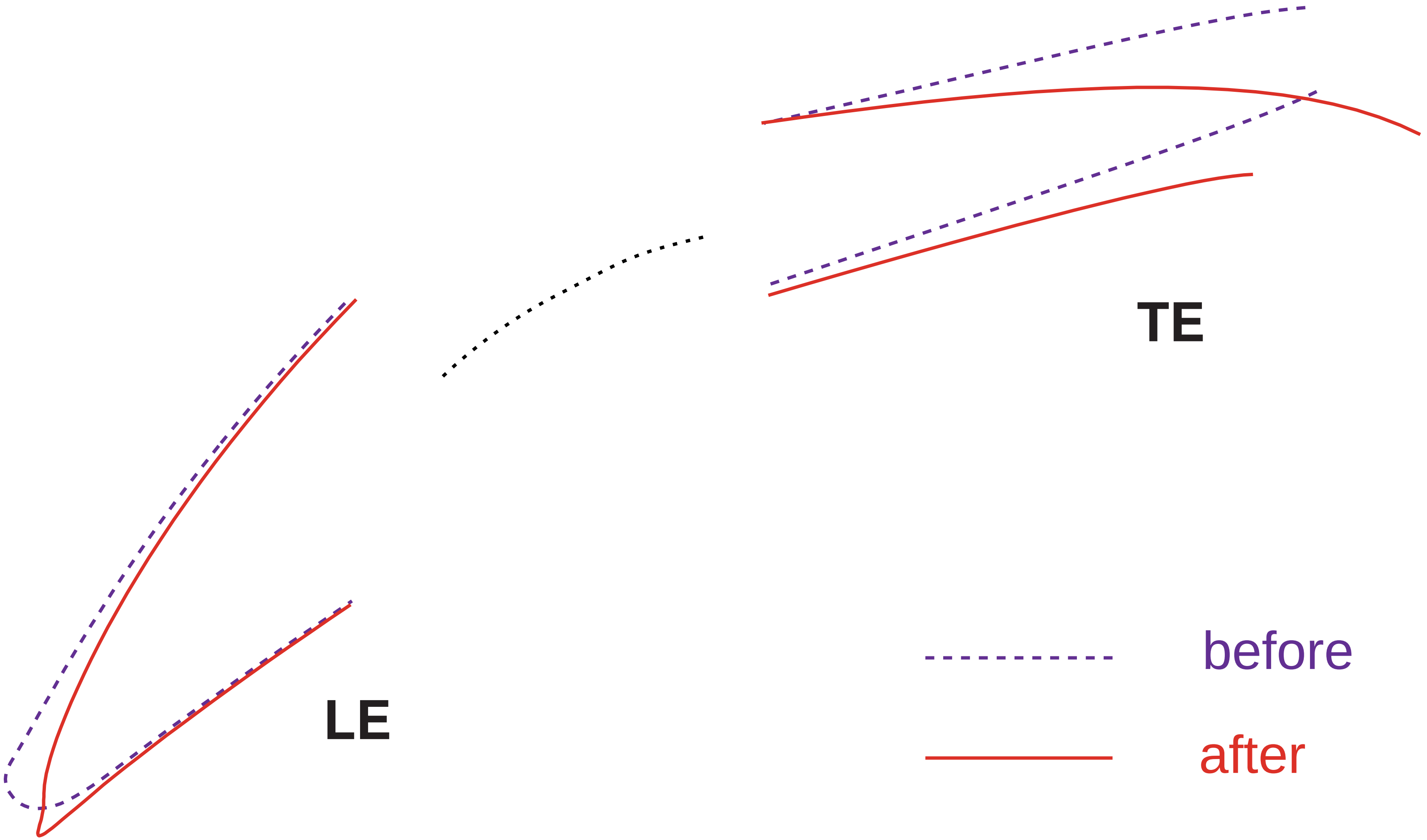
outlet flow angle:
18.9° --> 17.0°

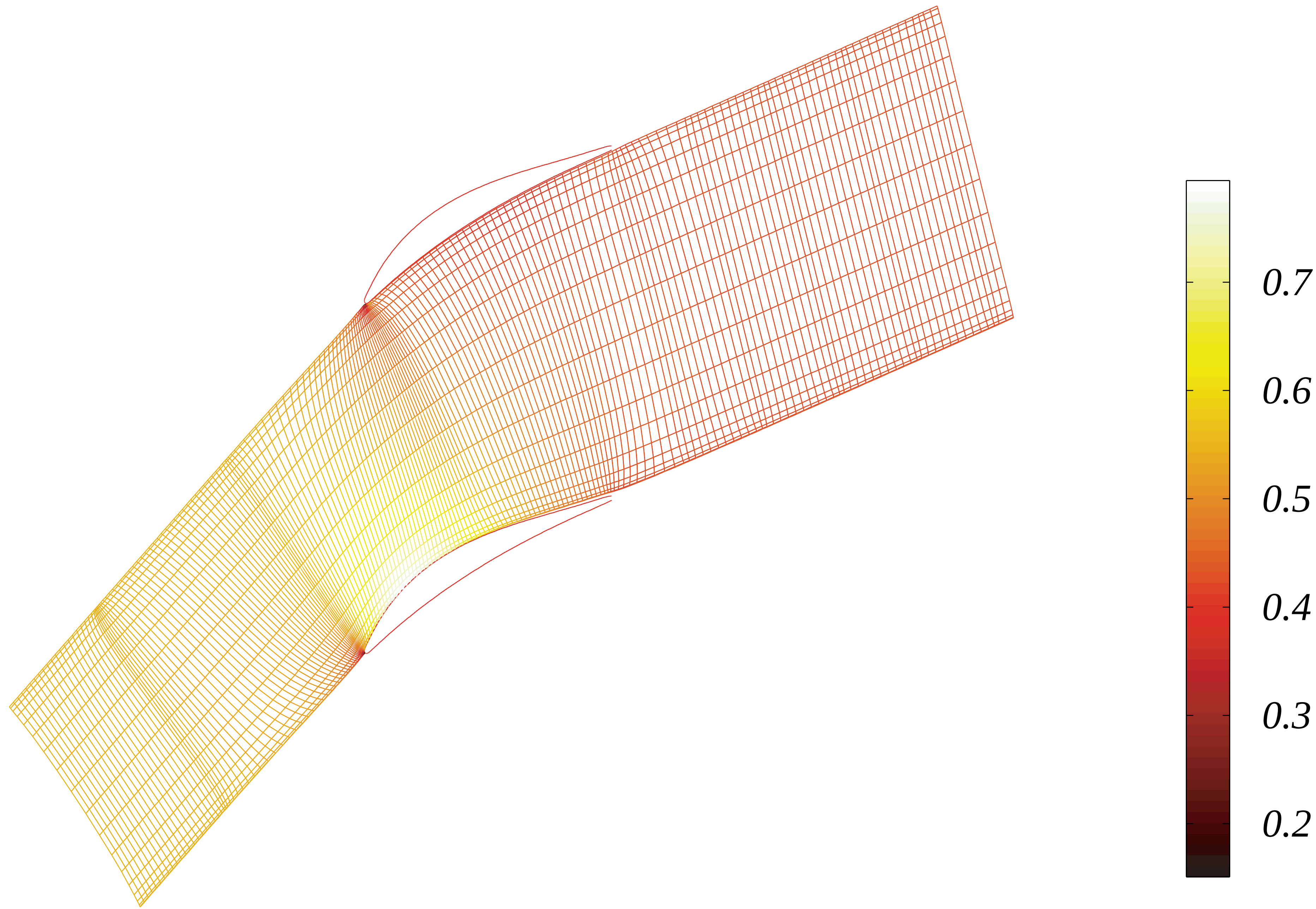
pseudo-constrained



outlet flow angle:
18.9° --> 19.2°

Optimization steps may well lead to crashes, e. g.




0.7
0.6
0.5
0.4
0.3
0.2

free stream: Euler equations with Coriolis- and centripetal forces

$$\int_{\partial A} \rho \, (\mathbf{v}^\top \boldsymbol{n}) \, ds \;=\; 0 \,,$$ *continuity equation*

$$\int_{\partial A} \big(\rho \, (\mathbf{v}^\top \boldsymbol{n}) \, \mathbf{v} \;+\; p \, \boldsymbol{n}\big) \, ds \;=\; -\int_A \rho \, \boldsymbol{F} \, dV \,,$$ *momentum equation*

$$\int_{\partial A} \rho \, R \, (\mathbf{v}^\top \boldsymbol{n}) \, ds \;=\; \mathbf{0} \,.$$ *energy equation*

boundary layer: equations for skin friction, displacement- and momentum-thickness

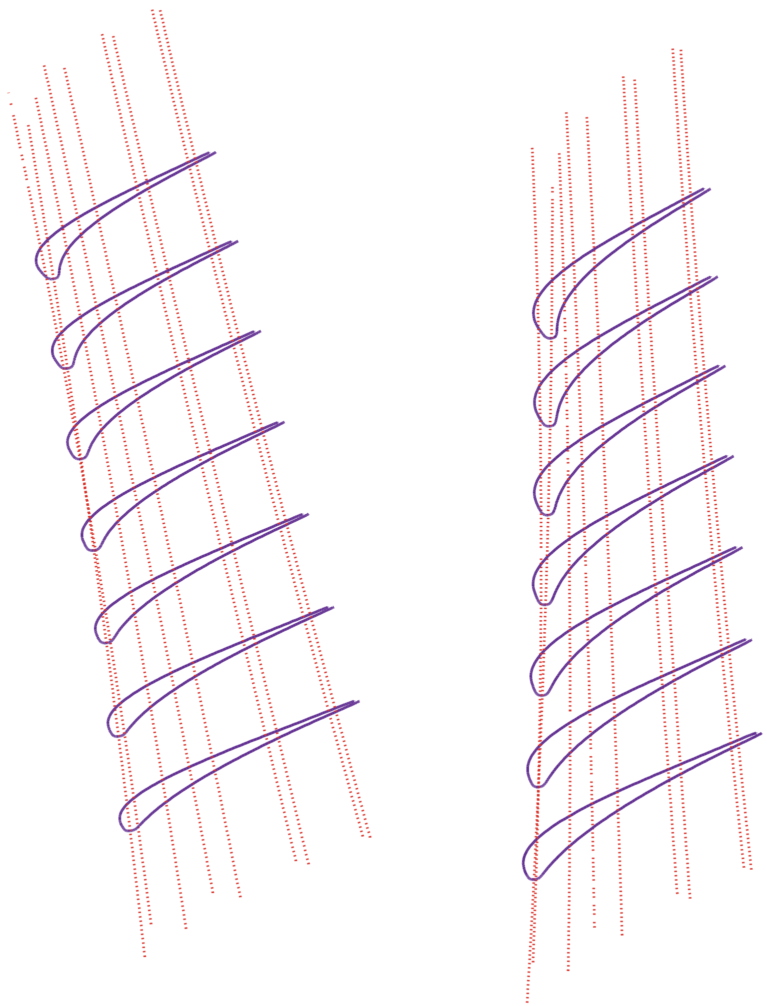
von Kármán momentum-integral equation $$\frac{d\theta}{ds} \;=\; \mathcal{F}_1(\theta, \delta^*, u_e) \,,$$

shape parameter equation $$\frac{dH^*(\theta, \delta^*, u_e)}{ds} \;=\; \mathcal{F}_2(\theta, \delta^*, u_e, C_\tau) \,,$$

Green's lag equation $$\frac{dC_\tau}{ds} \;=\; \mathcal{F}_3(\theta, \delta^*, u_e, C_\tau) \,.$$

The multiple setpoint problem structure arises from

+



Multiple setpoint optimization problem

$$\min_{\boldsymbol{x}_{11},\dots,\boldsymbol{x}_{NM},\boldsymbol{p}_1,\dots,\boldsymbol{p}_M} \sum_{j=1}^{M} \sum_{i=1}^{N} \omega_{ij}\, \mathrm{f}\left(\boldsymbol{x}_{ij}, \boldsymbol{p}_j, \alpha_i\right)$$

overall performance

$$\text{s. t.} \quad \begin{cases} \mathbf{c}(\boldsymbol{x}_{ij}, \boldsymbol{p}_j, \alpha_i) = \mathbf{0} \quad \forall\, i, j \\ \mathbf{g}_j(\boldsymbol{p}_j) \leq \mathbf{0} \quad \forall\, j \\ \mathbf{h}\left(\boldsymbol{p}_1, \dots, \boldsymbol{p}_M\right) \leq \mathbf{0} \end{cases}$$

discretized flow equations

streamsurface profile geometry constraints

3D-blade geometry constraints

OptiMISES PRSQP all-in-one slide

The profile parameter update $\Delta \boldsymbol{p}$ *is determined by*

$$\min_{\Delta \boldsymbol{p}} \quad \Delta \boldsymbol{p}^\top B \, \Delta \boldsymbol{p} + \gamma^\top \Delta \boldsymbol{p}$$

$$\text{s. t.} \quad \begin{pmatrix} \mathbf{g} \\ \mathbf{h} \end{pmatrix} + G \, \Delta \boldsymbol{p} \; \leq \; \mathbf{0}$$

$$G = \begin{pmatrix} \nabla_{\boldsymbol{p}_1}^\top \mathbf{g}_1 & & \\ & \ddots & \\ & & \nabla_{\boldsymbol{p}_M}^\top \mathbf{g}_M \\ \nabla_{\boldsymbol{p}_1}^\top \mathbf{h} & \dots & \nabla_{\boldsymbol{p}_M}^\top \mathbf{h} \end{pmatrix}$$

$$B = B\left(B^-, \Delta \boldsymbol{p}, \Delta \gamma^+, \rho\right)$$

$$\gamma = \nabla_{\boldsymbol{p}} \mathrm{f} - \nabla_{\boldsymbol{p}} \mathbf{c}^\top \, \nabla_{\boldsymbol{x}} \mathbf{c}^{-\top} \, \nabla_{\boldsymbol{x}} \mathrm{f}$$

and the flow variable update is

$$\Delta \boldsymbol{x} = \nabla_x \mathbf{c}^{-1} \left(\mathbf{c} + \nabla_p \mathbf{c} \, \Delta \boldsymbol{p} \right) \; .$$

The OptiMISES multiple setpoint reduced gradient

Since $\nabla_x \mathbf{c} = \begin{pmatrix} \nabla_{x_{1,1}} \mathbf{c}_{1,1} & & & & \\ & \ddots & & & \\ & & \nabla_{x_{N,1}} \mathbf{c}_{N,1} & & \\ & & & \ddots & \\ & & & & \nabla_{x_{N,M}} \mathbf{c}_{N,M} \end{pmatrix}$ is block-diagonal

the OptiMISES multiple setpoint reduced gradient

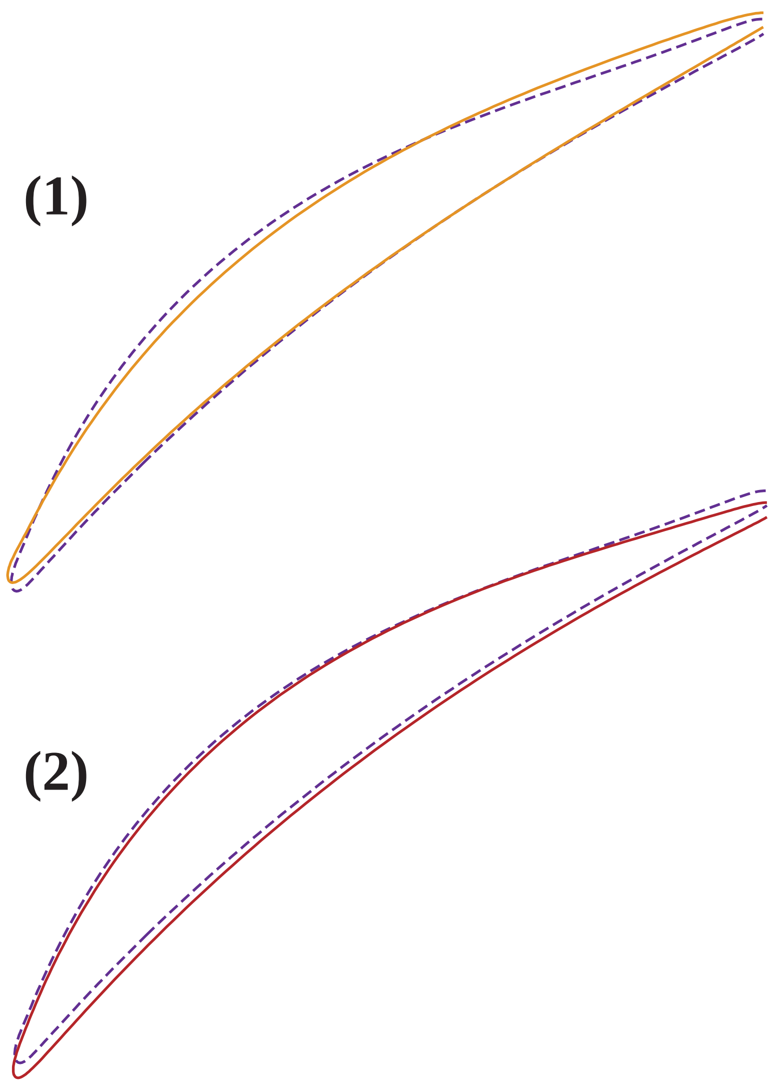
$$\boldsymbol{\gamma} = \nabla_{\boldsymbol{p}} \mathrm{f} - \nabla_{\boldsymbol{p}} \mathbf{c}^{\top} \nabla_{\boldsymbol{x}} \mathbf{c}^{-\top} \nabla_{\boldsymbol{x}} \mathrm{f}$$

has the form $\boldsymbol{\gamma} = \begin{pmatrix} \sum_{i=1}^{N} \omega_{i1} \boldsymbol{\gamma}_{i1} \\ \vdots \\ \sum_{i=1}^{N} \omega_{iM} \boldsymbol{\gamma}_{iM} \end{pmatrix}$

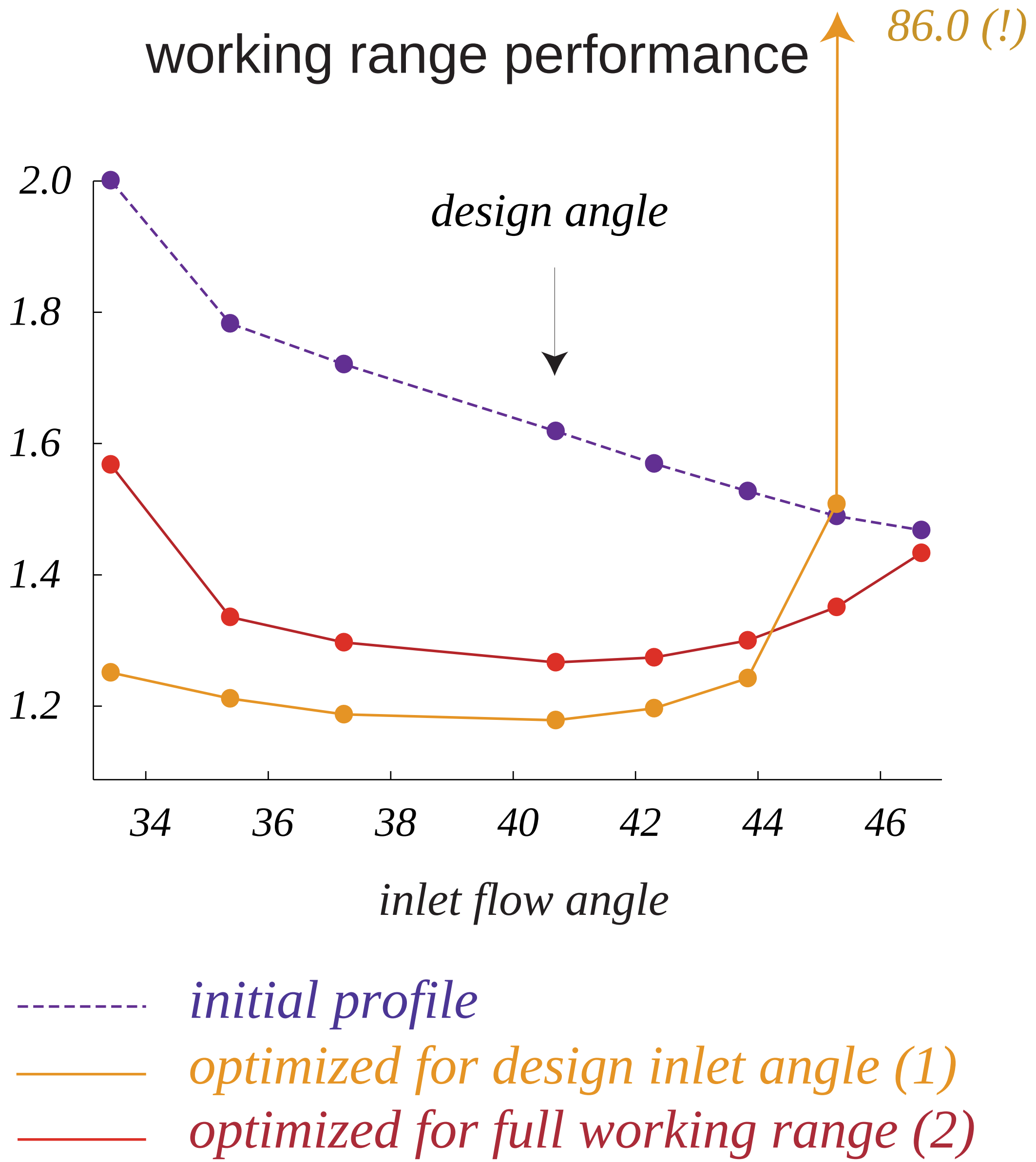
where $\boldsymbol{\gamma}_{ij} = \nabla_{\boldsymbol{p}_j} \mathrm{f}_{ij} - \nabla_{\boldsymbol{p}_j} \mathbf{c}_{ij}^{\top} \nabla_{\boldsymbol{x}} \mathbf{c}_{ij}^{-\top} \nabla_{\boldsymbol{x}} \mathrm{f}_{ij}$

Compressor blade working range example

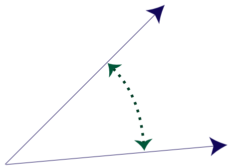
profile shapes



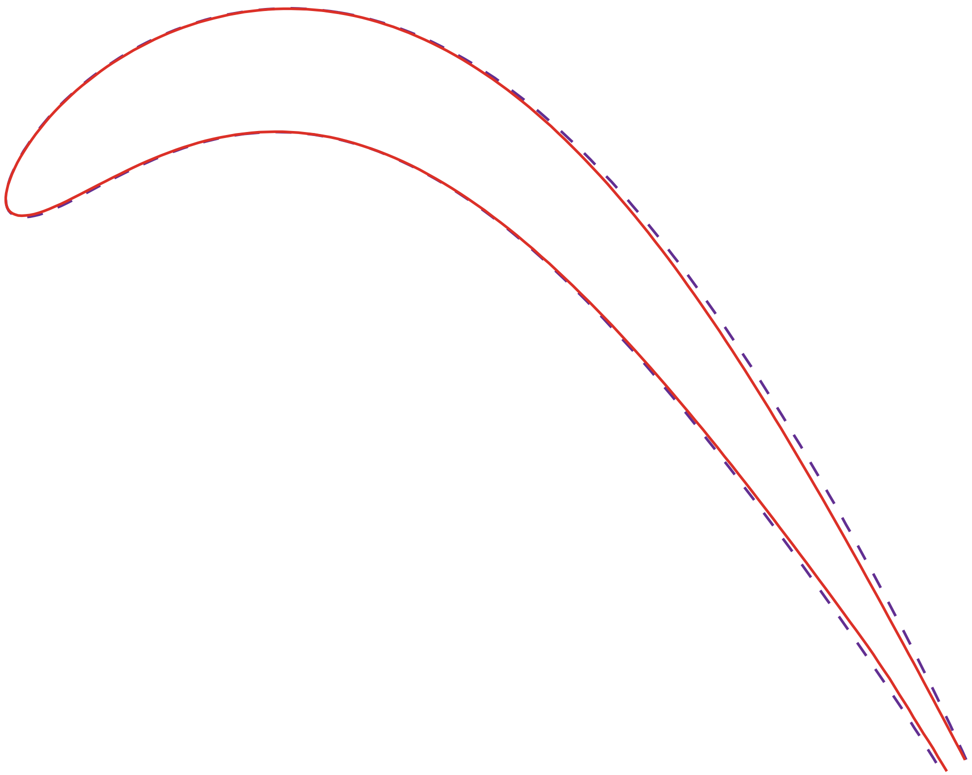
working range performance



Turbine blade working range example



working range
(inlet flow angle)



total pressure loss ζ (%)